# NUMERICAL APPROACHES FOR MODELING THE SWELLING BEHAVIOR OF HYDROGELS

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A sharp interface theory is developed for coherent phase transitions in synthetic hydrogels. In addition to the standard chemo-mechanical bulk and interfacial balance laws, the theory involves an ancillary interfacial constraint imposing configurational force balance. The key constitutive ingredient of the theory is an expression for the Gibbs-energy density as a function of the deformation gradient and the diffusion potential.

Motivated by the recent experiments of Olsen *et al.*[2], we study the swelling behavior of a cylindrical specimen that is confined in the transverse direction by two glass plates. A numerical approximation is then constructed using an adaptation of the methodologies presented in Ji *et al.*[1]. In the present approach, the standard FE bases are enriched with modified level-set functions enabling the numerical approximations to possess the correct degree of continuity across the interface. The level-set method is employed to model and evolve the phase interface given the velocity field. We present a new post-processing procedure for the accurate evaluation of the interfacial quantities such as the jump in the normal Eshelby stress and the jump in the solute flux.

Parametric studies are performed and the velocity profiles are compared to the observations of Olsen *et al.*[2]. Excellent qualitative agreement is demonstrated. The results are also compared to similar swelling studies of a spherical gel[3]. These studies suggest several possible synthetic pathways that might be pursued as a means to engineer hydrogels with optimal response times.

**References**

[1] H. Ji, D. Chopp, and J. Dolbow, "A hybrid extended finite element/level set method for modeling phase transformations," *International Journal for Numerical Methods in Engineering*, v. 51, p. 943-960, 2002.
[2] M.G. Olsen and J.M. Bauer, "Particle imaging technique for measuring the deformation rate of hydrogel microstructures," *Applied Physics Letters*, v. 76(22), p. 3310-3312, 2000.
[3] J. Dolbow, E. Fried, and H. Ji, "Chemically-induced swelling of hydrogels," submitted for publication, 2003.